C2

APPENDIX

Notation used throughout the flowcharts:

WCS	World Coordinate System
CCS	C-arm Coordinate System
(x, y, z)	Used for 3D coordinates in WCS and the CCS.
(x, y)	Used for calibrated image coordinates.
(u, v)	Used for real image coordinates.
α	Sagittal Angle.
β	Transverse Angle.
γ	Approach Angle.

Subscripts:

w = WCS c = CCS	Specifies the coordinate system. Only used with 3D coodinate systems.
t = top b = bottom	Specifies a point on the virtual guidewire.
a = A/P s = Sagittal	Specifies to what image the information pertains to.

[1] J. Canny; "A Computational Approach to Edge Detection"; IEEE Transactions on Pattern Analysis Machine Intelligence; Vol 8, Nov. 1986, pp. 679-698.

[2] Mathematics involved in performing the Levenberg-Marquardt optimization method:

$$R = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi \end{bmatrix}$$

$$u_c(x_i; a) = \left(\frac{R_{11}x + R_{12}y + R_{13}z + t_x}{R_{31}x + R_{32}y + R_{33}z + t_z}\right) f$$

and

$$v_c(x_i; a) = \left(\frac{R_{21}x + R_{22}y + R_{23}z + t_y}{R_{31}x + R_{32}y + R_{33}z + t_z}\right) f$$

$$\chi^2(x; a) = \sum_{i=0}^{8}((u_i - u_c(x_i; a))^2 + (v_i - v_c(x_i; a))^2)$$

where $x_i = [x, y, z]_i$ are the 3D coordinates of the fiducials, (u, v) are the 2D coordinates of the center of the fiducials, and $a = [\phi, \theta, \psi, t_x, t_y, t_z]$ are the six parameters that define a six degree-of-freedom pose.

[3] Once the fit has been performed I construct the homogeneous transformation matrix that corresponds to the optimized parameters ($a = [\phi, \theta, \psi, t_x, t_y, t_z]$) as follows:

$$REG_A = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & px \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & py \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi & pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[4] Once the fit has been performed I contruct the homogeneous transformation matrix that corresponds to the optimized parameters ($a = [\phi, \theta, \psi, t_x, t_y, t_z]$) as follows:

$$\mathbf{REG_B} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & px \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & py \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi & pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[5] The line of sight is calculated in the following way:
The line of sight is bound by (0, 0, 0) and (u_c, v_c, f) in the CCS.
Note: (u_c, v_c) is the calibrated equivalent of (u, v). See [13]

$$\begin{bmatrix} LSx_{w1} & LSx_{w2} \\ LSy_{w1} & LSy_{w2} \\ LSz_{w1} & LSz_{w2} \\ 1 & 1 \end{bmatrix} = [\mathbf{REG_A}]^{-1} \begin{bmatrix} u_c & 0 \\ v_c & 0 \\ f & 0 \\ 1 & 1 \end{bmatrix}$$

C2 cont.

$$\begin{bmatrix} x_{cs1} & x_{cs2} \\ y_{cs1} & y_{cs2} \\ z_{cs1} & z_{cs2} \\ 1 & 1 \end{bmatrix} = [\mathbf{REG_A}] \begin{bmatrix} LSx_{w1} & LSx_{w2} \\ LSy_{w1} & LSy_{w2} \\ LSz_{w1} & LSz_{w2} \\ 1 & 1 \end{bmatrix}$$

$$u_1 = \frac{x_{cs1}}{z_{cs1}} f \qquad v_1 = \frac{y_{cs1}}{z_{cs1}} f$$

$$u_2 = \frac{x_{cs2}}{z_{cs2}} f \qquad v_2 = \frac{y_{cs2}}{z_{cs2}} f$$

Due to the inherent distortion in the fluoroscopic images the line of sight is drawn as a curve image. This is done by *un-calibrating* 50 points on the line bound by (u_1, v_1) and (u_2, v_2) as in [15] and drawing a polyline through them.

[6] Recall that the virtual guidewire is a 3D object bound by (0_{wt}, 0_{wt}, 0_{wt}) and (0_{wb}, 0_{wb}, $-screwlength_{wb}$).
$\beta = \beta + 0.1$ * (# pixels moved by the trackball)

$$\begin{bmatrix} Vx_{wt} & Vx_{wh} \\ Vy_{wt} & Vy_{wh} \\ Vz_{wt} & Vz_{wh} \\ 1 & 1 \end{bmatrix} = [\mathrm{T}^{1}(\alpha, \beta, tx, ty, tz)] \begin{bmatrix} 0_{wt} & 0_{wb} \\ 0_{wt} & 0_{wb} \\ 0_{wt} & -screwlength_{wb} \\ 1 & 1 \end{bmatrix}$$

[7] With (Vx_{wt}, Vy_{wt}, Vz_{wt}) and (Vx_{wh}, Vy_{wb}, Vz_{wh}) the virtual guidewire's projection is drawn on both the A/P and sagittal images using the following equations:

$$\begin{bmatrix} x_{cat} & x_{cab} \\ y_{cat} & y_{cab} \\ z_{cat} & z_{cab} \\ 1 & 1 \end{bmatrix} = [\mathbf{REG_A}] \begin{bmatrix} Vx_{wt} & Vx_{wb} \\ Vy_{wt} & Vy_{wh} \\ Vz_{wt} & Vz_{wh} \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{cst} & x_{csb} \\ y_{cst} & y_{csh} \\ z_{cst} & z_{csh} \\ 1 & 1 \end{bmatrix} = [\mathbf{REG_S}] \begin{bmatrix} Vx_{wt} & Vx_{wh} \\ Vy_{wt} & Vy_{wh} \\ Vz_{wt} & Vz_{wh} \\ 1 & 1 \end{bmatrix}$$

APPENDIX

$$u_{at} = \frac{x_{cat}}{z_{cat}} f \qquad v_{at} = \frac{y_{cat}}{z_{cat}} f$$

$$u_{st} = \frac{x_{cst}}{z_{cst}} f \qquad v_{st} = \frac{y_{cst}}{z_{cst}} f$$

$$u_{ab} = \frac{x_{cab}}{z_{cab}} f \qquad v_{ab} = \frac{y_{cab}}{z_{cab}} f$$

$$u_{sb} = \frac{x_{csb}}{z_{csb}} f \qquad v_{sb} = \frac{y_{csb}}{z_{csb}} f$$

Due to the distortion in fluoroscopic images the projected guidewire is drawn as a curve. This is done by *un-calibrating* 20 points on the line bound by (u_{at}, v_{at}) and (u_{ab}, v_{ab}) as in [15] and drawing a polyline through them on the A/P image and similarly for the Sagittal image using (u_{st}, v_{st}) and (u_{sb}, v_{sb}).

[8] To draw the virtual guidewire's projection, two points (0, 0, 0) and (0, 0, *-screwlength*), in the WCS are transformed so that the top point (0, 0, 0) lies on the line of sight. The virtual guidewire is initially set to 30mm. The projected guidewire is drawn using the following math:

C2 Cont.

initially:
$depth = 0.2$
$screwlength = 30\text{mm}$
$\alpha = 0,\ \beta = 0$
(tx, ty, tz) is constrained to lie on the line of sight bound by $(LSx_{w1}, LSy_{w1}, LSz_{w1})$ and $(LSx_{w2}, LSy_{w2}, LSz_{w2})$, thus

$$tx = LSx_{w1} - depth*(LSx_{w2} - LSx_{w1})$$
$$ty = LSy_{w1} - depth*(LSy_{w2} - LSy_{w1})$$
$$tz = LSz_{w1} - depth*(LSz_{w2} - LSz_{w1})$$

$$\begin{bmatrix} Vx_{wt} & Vx_{wb} \\ Vy_{wt} & Vy_{wb} \\ Vz_{wt} & Vz_{wb} \\ 1 & 1 \end{bmatrix} = [\,T^{1}(\alpha, \beta, tx, ty, tz)] \begin{bmatrix} 0_{wt} & 0_{wb} \\ 0_{wt} & 0_{wb} \\ 0_{wt} & -screwlength_{wb} \\ 1 & 1 \end{bmatrix}$$

in order to draw the projected guidewire on the images, the points $(Vx_{wt}, Vy_{wt}, Vz_{wt})$ and $(Vx_{wb}, Vy_{wb}, Vz_{wb})$ are used in conjunction [7].

[9] Recall that the virtual guidewire is a 3D object bound by $(0_{wt}, 0_{wt}, 0_{wt})$ and $(0_{wb}, 0_{wb}, -screwlength_{wb})$.

$depth = depth + 0.1$ * (# pixels moved by the trackball)

$$tx = LSx_{w1} - depth*(LSx_{w2} - LSx_{w1})$$
$$ty = LSy_{w1} - depth*(LSy_{w2} - LSy_{w1})$$
$$tz = LSz_{w1} - depth*(LSz_{w2} - LSz_{w1})$$

---

[1] **T** is composed of the following transformations:
T = Trans(tx, ty, tz) Rot(y, α) Rot(x, β)
or

$$[\,T\ (\alpha, \beta, tx, ty, tz)] = \begin{bmatrix} \cos\alpha & \sin\alpha\sin\beta & \sin\alpha\cos\beta & tx \\ 0 & \cos\beta & -\sin\beta & ty \\ -\sin\alpha & \cos\alpha\sin\beta & \cos\alpha\cos\beta & tz \end{bmatrix}$$

APPENDIX

$$\begin{bmatrix} Vx_{wt} & Vx_{wh} \\ Vy_{wt} & Vy_{wh} \\ Vz_{wt} & Vz_{wh} \\ 1 & 1 \end{bmatrix} = [T^1(\alpha, \beta, tx, ty, tz)] \begin{bmatrix} 0_{wt} & 0_{wh} \\ 0_{wt} & 0_{wh} \\ 0_{wt} & -screwlength_{wh} \\ 1 & 1 \end{bmatrix}$$

[10] Recall that the virtual guidewire is a 3D object bound by (0_{wt}, 0_{wt}, 0_{wt}) and (0_{wh}, 0_{wh}, $-screwlength_{wh}$).

$\alpha = \alpha + 0.1$ * (# pixels moved by the trackball)

$$\begin{bmatrix} Vx_{wt} & Vx_{wh} \\ Vy_{wt} & Vy_{wh} \\ Vz_{wt} & Vz_{wh} \\ 1 & 1 \end{bmatrix} = [T^1(\alpha, \beta, tx, ty, tz)] \begin{bmatrix} 0_{wt} & 0_{wh} \\ 0_{wt} & 0_{wh} \\ 0_{wt} & -screwlength_{wh} \\ 1 & 1 \end{bmatrix}$$

[11] Recall that the virtual guidewire is a 3D object bound by (0_{wt}, 0_{wt}, 0_{wt}) and (0_{wb}, 0_{wb}, $-\boldsymbol{screwlength}_{wh}$).

C2
Cont.

$$\begin{bmatrix} Vx_{wt} & Vx_{wb} \\ Vy_{wt} & Vy_{wb} \\ Vz_{wt} & Vz_{wb} \\ 1 & 1 \end{bmatrix} = [T^1(\alpha, \beta, tx, ty, tz)] \begin{bmatrix} 0_{wt} & 0_{wh} \\ 0_{wt} & 0_{wb} \\ 0_{wt} & -\boldsymbol{screwlength}_{wb} \\ 1 & 1 \end{bmatrix}$$

[12] Given

$$[\mathbf{T}_{ool}]^2 = [\mathbf{Rot}(z, -90)][\mathbf{Rot}(y, -90)]$$
$$[\mathbf{P}_{lan}]^2 = [\mathbf{Rot}(y,\alpha)][\mathbf{Rot}(x,\beta)][\mathbf{T}_{ool}]$$
$$[\mathbf{A}_{pproach}]^2 = [\mathbf{Rot}(z,\gamma)]$$
$$[\mathbf{F}_{inal}\mathbf{P}_{lan}]^2 = \begin{bmatrix} P_{Nx} & & \\ P_{Ny} & ? & ? \\ P_{Nz} & & \end{bmatrix}$$

and using the following contraints I determine the remaining two vectors that would be complete **[FP]**. Note: The first vector (N) is maintained from the **[Plan]** since it is the dirll guide axis:

Contraints:

1) $FP_{Ax}^2 + FP_{Ay}^2 + FP_{Az}^2 = 1$
2) $A_N \cdot FP_A = 0$
3) $FP_N \cdot FP_A = 0$

$$D = -\frac{A_{Ny}\,(A_{Nx} \cdot FP_{Nz} - FP_{Nx} \cdot A_{Nz})}{A_{Nx}\,(FP_{Nx} \cdot A_{Ny} - A_{Nx} \cdot FP_{Ny})} - \frac{A_{Nz}}{A_{Nx}}$$

$$E = \frac{(A_{Nx} \cdot FP_{Nz} - FP_{Nx} \cdot A_{Nz})}{(FP_{Nx} \cdot A_{Ny} - A_{Nx} \cdot FP_{Ny})}$$

---

[2] These matrices are of the following form:

$$\begin{bmatrix} Nx & Ox & Ax \\ Ny & Oy & Ay \\ Nz & Oz & Az \end{bmatrix}$$

$FP_{Az} = \pm\sqrt{D^2 + E^2 + 1}$
$FP_{Ax} = D \cdot FP_{Az}$
$FP_{Ay} = E \cdot FP_{Az}$

FP_O is determined using

$FP_O = FP_N \times FP_A$

Hence.

$$[\mathbf{F}_{\text{inal}}\mathbf{P}_{\text{lan}}] = \begin{bmatrix} P_{Nx} & FP_{Ox} & FP_{Ax} \\ P_{Ny} & FP_{Oy} & FP_{Ay} \\ P_{Nz} & FP_{Oz} & FP_{Az} \end{bmatrix}$$

Since the PUMA 560 robot uses an Euler representation for specifying an orientation, the inverse solution of **[FP]** is determined in the following manner:

Euler representation = Rot(z,ϕ) Rot(y, θ) Rot(z, ψ) thus from [i].

C2 cont.

$\phi = \arctan(FP_{Ay}, FP_{Ax})$
$\theta = \arctan(FP_{Ax} \cdot \cos(\phi) + FP_{Ay} \cdot \sin(\phi), FP_{Az})$
$\psi = \arctan(-FP_{Nx} \cdot \sin(\phi) + FP_{Ny} \cdot \cos(\phi), -FP_{Ox} \cdot \sin(\phi) + FP_{Oy} \cdot \cos(\phi))$

Adding a PUMA specific offset to ϕ, and θ the final position and orientation is established

Final pose = (ϕ + 90, θ - 90, ψ, tx, ty, tz)

[13] The calibrated coordinates (x, y) of the edge-pixels (u, v) are determined using a quartic polynomial equation as follows:

$x = a_0\, u^4v^4 + a_1\, u^4v^3 + a_2\, u^4v^2 + \ldots + a_{23}\, uv + a_{24}$
$y = a_0\, u^4v^4 + a_1\, u^4v^3 + a_2\, u^4v^2 + \ldots + a_{23}\, uv + a_{24}$

the set of parameters a and b, are previously determined using the image calibration program.

[14] The center of the fiducial shadow is found by fitting the equation of a circle to the edge-pixels using a pseudo-inverse approach:

$$\begin{bmatrix} x^2_0 + y^2_0 \\ \vdots \\ x^2_n + y^2_n \end{bmatrix} = \begin{bmatrix} x_0 & y_0 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} 2h \\ 2k \\ r^2 - h^2 - k^2 \end{bmatrix}$$

or

$\mathbf{A} = \mathbf{BP}$

using pseudo inverse

$\mathbf{P} = (\mathbf{B}^T\mathbf{B})^{-1}\, \mathbf{B}^T\mathbf{A}$

once **P** is established the center of the fiducials (h, k) is determined as follows:

$$h = \frac{p_0}{2}$$

$$k = \frac{p_1}{2}$$

[15] The un-calibrated (distorted) coordinates (u, v) corresponds to the calibrated coordinate (x, y) and is determined using a quartic polynomial equation as follows:

C2 Cont

$$u = a_0\, x^4 y^4 + a_1\, x^4 y^3 + a_2\, x^4 y^2 + \ldots + a_{23}\, xy + a_{24}$$
$$v = a_0\, x^4 y^4 + a_1\, x^4 y^3 + a_2\, x^4 y^2 + \ldots + a_{23}\, xy + a_{24}$$

the set of parameters a and b. are previously determined using a separate calibration program.

---

i Robot Manipulators: Mathematics, Programming, and Control; Richard P. Paul; The MIT Press, Cambridge, Massachusetts and London, England, 1983.